Causal inference under Interference (joint with Sohom Bhattacharya)

Setup: Binary treatment, $T \in \{\pm 1\}$.

n-individuals. $(Y_i(+1), Y_i(-1))$ – Potential outcomes.

Challenge: Observe $y_i = Y_i(T_i)$!

Target: $\tau = \frac{1}{n} \sum_{i=1}^{n} \mathbb{E}[Y_i(+1) - Y_i(-1)]$ (ATE)

Exp design: $T \perp\!\!\!\perp \{(Y_i(+1), Y_i(-1)) : 1 \le i \le n\}$

$$\hat{\tau} = \bar{y}\big|_{T=1} - \bar{y}\big|_{T=-1}.$$

Obs study $\to$ challenging!

Today: Interference. Potential outcomes $\{Y_i(t) : t \in \{\pm 1\}^n\}$.

Motivation: Online experiments.

Causal effects:

$$DE_i(t_{-i}) = \mathbb{E}[Y_i(1, t_{-i})] - \mathbb{E}[Y_i(-1, t_{-i})]$$

$$DE = \frac{1}{n} \frac{1}{2^{n-1}} \sum_{i=1}^{n} \sum_{t_{-i}} DE_i(t_{-i}) \quad \text{[Direct effect]}$$

$$IE(t_{-i}) = \mathbb{E}[Y_i(-1, t_{-i})] - \mathbb{E}[Y_i(-1)]$$

$$IE = \frac{1}{n 2^{n-1}} \sum_{i=1}^{n} \sum_{t_{-i}} IE_i(t_{-i}).$$

Task: Estimate DE, IE from observed data.

Data: (y_i, T_i, X_i) $y_i \in \mathbb{R}$, $T_i \in \{\pm 1\}$, $X_i \in \mathbb{R}^d$

Assumption (i) $y_i = Y_i(T)$ (Consistency)

(ii) $\{Y_i(t) : 1 \le i \le n, t \in \{\pm 1\}^n\} \perp\!\!\!\perp T \mid X$

(No unmeasured confounding)

(iii) $\mathbb{P}[T = t] \geq \sigma_n > 0 \ \forall \ t \in \{\pm 1\}^n$. (positivity)

Can show: Under A1-A3, DE & IE are fns of the law of $\{(y_i, T_i, X_i) : 1 \le i \le n\}$

Need: To escape from multi-verse!

Structure on $\{Y_i(t) : 1 \le i \le n\}$

$$\mathbb{P}[Y_i(t) = y] \propto \exp\left(\frac{1}{2} y^T A y + \tau_0 \sum_{i=1}^n t_i y_i + \sum_{i=1}^n y_i \langle x_i, \theta_0 \rangle\right) \prod_{i=1}^n d\mu(y_i)$$

$\mu \in \mathcal{P}r([-1,1])$

Assume A is known. (Following (Tchetgen-Tchetgen, Schpitser, Fulcher))

$+ X_i \sim \mathbb{P}_X$ iid on $[-1,1]^d$.

Questions

(i) Estimate τ_0, θ_0 from observed data.

(ii) Compute DE given τ_0, θ_0.

Lemma: $DE = \frac{2}{n} \sum_{i=1}^n \mathbb{E}_{\substack{T \sim Unif(\pm 1) \\ X \sim \mathbb{P}_X}} [T_i \langle Y_i \rangle]$.

Idea: Estimate $\langle Y \rangle$ & use plug in estimate.

__Theorem.__ Suppose ~~through~~ either

(i) $\mathrm{Tr}(A^2) = o(n)$, $\|A\| = O(1)$ or

(ii) $A \sim \mathrm{GOE}(1/n)$

then $\langle Y \rangle$ can be estimated efficiently if $\|A\| < C_0$ (high-temperature regime)

Let $\widehat{\langle y \rangle} = m$ & use $\widehat{DE} = \frac{2}{n} \sum_{i=1}^{n} T_i m_i$.

Then $\left| \mathbb{E}_{T,X}\left[\widehat{DE}\right] - DE \right| \to 0$.

__Rmk:__ Need to repeat alg with distinct samples.

__Parameter Estimation:__

__Idea:__ Use pseudo-likelihood.

$$(\hat{\tau}_0, \hat{\theta}_0) = \underset{\tau_0, \theta_0}{\arg\max} \prod_{i=1}^{n} \mathbb{P}(y_i \mid y_{-i}, T_i, X_i)$$

__Thm:__ Assume $\mathbb{P}(T = t) \propto \exp\left(\frac{1}{2} t^T M t + \sum_i t_i \langle x_i, \eta_0 \rangle\right)$

$\|A\| = O(1)$, $\|M\| = O(1)$. Then $\cancel{\sqrt{n}(\hat{\tau}_0, \hat{\theta}_0)}$

$(\hat{\tau}_0, \hat{\theta}_0)$ is $\sqrt{n}$-consistent.

$\to$ Simulations.

__Open questions.__ (i) Low temp? Stat/Comp gaps.

(ii) What if A is unobserved?